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On vanishing two loop cosmological constants in nonsupersymmetric strings

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Abstract: It has recently been suggested that in certain special nonsupersymmetric type II string compactifications, at least the first two perturbative contributions to the cosmological constant Λ vanish. Support for perturbative vanishing beyond 1-loop (as well as evidence for the absence of some nonperturbative contributions) has come from duality arguments. There was also a direct 2-loop computation which was incomplete; in this note we explain the deficiency of the previous 2-loop calculation and discuss the complete 2-loop computation in two different models. The corrected analysis yields a vanishing 2-loop contribution to Λ in these models.

Keywords: Superstring Vacua, Superstrings and Heterotic Strings, Supersymmetry Breaking.

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Contents

1. Introduction

A new class of non-supersymmetric type II models was recently introduced [1, 2]. These models have a simple mechanism for cancellation of the 1-loop vacuum energy (due to a guaranteed bose-fermi degeneracy). A similar class of models was constructed in the free-fermionic prescription in [3]. In [1] a heuristic argument suggesting that the cancellation should persist to all higher loops was presented. A broader motivation arising from the correspondence between quantum-mechanical fixed lines and flat dilaton potentials [4] in the AdS/CFT correspondence [5] also suggests that cancellation should occur. Various string duality computations now provide indirect evidence for the absence of certain perturbative (and also certain nonperturbative) contributions to Λ in these models [6, 2, 3].

In [1] a direct analysis of the 2-loop contribution to the cosmological constant was also presented. It was argued that, in a particular gauge, it cancels pointwise on the moduli space. This was based on an analysis of only the spin-structure-dependent part of the amplitude. This analysis was incomplete. In the gauge we chose, picture changing operators coincide, and their operator product expansion produces poles. Some of these pole pieces cancel between matter and ghost contributions, or constitute total derivatives on the moduli space. In general, however, there can be finite pieces which arise from the Taylor expansion of the spin-structure dependent piece.[1]

In this note we rectify this situation for both the original model of [1] and for an even simpler model which also enjoys similar properties, and was introduced in [2].

[1] We thank Greg Moore for pointing out this error and for providing helpful constructive comments.

This analysis will also appear in a revised version of [1]. In the simpler model, if we choose the gauge with coincident picture-changing operators, summing over *all* spin structures produces a high enough order zero in the spin-structure-dependent piece to cancel against surviving poles in the spin-structure-independent factor. We discuss this cancellation in §4. In §5, we analyze the Taylor expansion of the original set of models [1]. We show that after summing over spin structures all potentially finite terms either cancel between different terms in the correlator of picture changing operators or vanish directly.

Perturbative string calculations at higher genus are notoriously subtle. In all of our analysis we assume that the prescription [7] (following e.g. [8] and [9]) for the supersymmetric 2-loop GSO phases is correct (in particular, in [10] it was argued that this prescription leads to the correct phases upon factorization). Adjusting the supersymmetric expression to incorporate the phases and twists of our nonsupersymmetric models is relatively straightforward, and we see that the cancellation persists if we make a further specialization of the gauge choice. Different gauge choices differ by total derivatives on the moduli space as explained in [11, 12]. This argument applies twist structure by twist structure in the orbifold theory, as what it uses is deformation of a contour integral of the BRST charge, which is invariant under the orbifold group. The corresponding boundary contributions cancel as discussed in [1]. As explained in [1], issues of modular invariance of the gauge choice are simpler in our models due to shifts involved in the orbifold group elements, which greatly reduce the modular group in a given twist structure.

2. The simpler model

S^1	F	G
1	1	$(s,-1)$
2	1	$(s,-1)$
3	1	$(s,-1)$
4	1	$(s,-1)$
5	1	$(0,s^2)$
6	(s,s)	1
	$(-1)^{F_R}$	$(-1)^{F_L}$

In addition to the original model of [1], we will also be considering a new model introduced in [2]. Since the description of the new model there is somewhat terse, we introduce it in greater detail here. Like the model of [1], it is an orbifold generated by two elements F and G. We start from the type II string compactified on a T^6 which is a product of six circles at the self-dual radius $R = l_s/\sqrt{2}$, where $l_s = \sqrt{\alpha'}$ is the string scale. F and G act on the left and right moving degrees of freedom of the superstring as s refers to a shift by $R/2 = l_s/2\sqrt{2}$. As explained in [2], this model has vanishing 1-loop vacuum energy (after integrating over Re τ) due to the fact that the trace of G acting on level matched states in the F-twisted sector vanishes,[2] and the other modular inequivalent contributions vanish by zero modes due to effective supersymmetry.

[2]The relevant fundamental domain for the integral is the strip instead of the normal keyhole region due to the shifts in G.

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3. 2-loops with supersymmetry

In this section we review the supersymmetric cancellation at 2 loops. As explained for example in [13, 11, 12], the type II string path integral can be written as

$$\int_{\mathcal{SM}_h} d\mu_0 \int [dBdCdX] e^{-S} \prod_{r=1}^{6h-6} (\eta_r, B) \prod_{a=1}^{4h-4} \delta((\eta_a, B)) \,, \tag{3.1}$$

Here B, C denote the b, β and c, γ ghosts, where (b, c) are the spin-(2,-1) conformal ghosts and (β, γ) are the spin-(3/2,-1/2) superconformal ghosts. X denotes the matter fields and η_r and η_a are Beltrami differentials relating the metric and gravitino to the moduli and supermoduli. In components,

$$(\eta_r, B) = \int \eta_{r\bar{z}}^{z} b_{zz} + \int \eta_{r\bar{z}}^{+} \beta_{z+} + \int \eta_{rz}^{\bar{z}} b_{\bar{z}\bar{z}} + \int \eta_{rz}^{-} b_{\bar{z}-} \,, \tag{3.2}$$

$$(\eta_a, B) = \int \eta_{a\bar{z}}^{z} b_{zz} + \int \eta_{a\bar{z}}^{+} \beta_{z+} + \int \eta_{az}^{\bar{z}} b_{\bar{z}\bar{z}} + \int \eta_{az}^{-} b_{\bar{z}-} \,. \tag{3.3}$$

As explained e.g. in [14, 13], we can write the path integral measure on supermoduli space in terms of a fixed measure on moduli space

$$d\mu_0 = d\mu [sdet(\eta, \Phi)]^{-1} [sdet(\Phi, \Phi)]^{1/2} \,, \tag{3.4}$$

Here $d\mu$ is a fixed measure on the supermoduli space $\mathcal{SM}_h$, integrated over a fixed domain independent of the beltrami differentials. Φ contains the $3h-3$ holomorphic and $3h-3$ antiholomorphic 2-differentials (b ghost zero mode wavefunctions) and the $2h-2$ holomorphic and $2h-2$ antiholomorphic 3/2-differentials (β ghost zero modes).

After choosing delta-function support for the worldsheet gravitinos, and integrating out the supermoduli, one obtains a correlation function of picture changing operators [11]

$$: e^{\phi} T_F := c\partial\xi + \frac{1}{2} e^{\phi} \psi^{\mu} \partial X^{\mu} - \frac{1}{4} \partial\eta e^{2\phi} b - \frac{1}{4} \partial \left(\eta e^{2\phi} b \right) \,, \tag{3.5}$$

and other ghost insertions

$$\sum_{\alpha,\beta,twists} \int d\mu [sdet(\eta, \Phi)]^{-1} [sdet(\Phi, \Phi)]^{1/2} [dX][dB][dC] e^{-S} (\hat{\eta}, b)^{6h-6} \xi(x_0) \times$$

$$\times \prod_{a=1}^{2h-2} : e^{\phi} T_F(z_a) : \prod_{a=2h-3}^{4h-4} : e^{\bar{\phi}} \bar{T}_F(z_a) : \bar{\xi}(\bar{y}_0) \,, \tag{3.6}$$

The superconformal ghosts $\beta = \partial\xi e^{-\phi}$, $\gamma = \eta e^{\phi}$ are defined in terms of spin-0 and spin -1 fermions ξ, η and a scalar ϕ [15]. The spin-0 fermion ξ has a zero mode on the surface which is absorbed by the insertion of $\xi(x_0)$ in (3.6). There is an anomaly in the ghost number $U(1)$ current which requires insertions of operators with total ghost number $2h-2$ to get a nonvanishing result. The correlation functions (3.6) can be evaluated using the formulas derived in e.g. [16, 17].

We will be interested in a gauge choice for which ultimately $z_1, z_2 \to \Delta_\gamma$, where Δ_γ is a divisor corresponding to an odd spin structure γ. We need to consider the (left-moving) spin-structure-dependent pieces of the correlation function, the poles arising from the spin-structure-independent local behavior of the picture changing correlator, and the behavior of the determinant (3.4) in this gauge. According to [11] we have the following contributions to the spin-structure-dependent pieces of the 2-loop partition function. The matter part of T_F contributes

$$\sum_\delta \langle\delta|\gamma\rangle \frac{\theta[\delta]^4(0)\theta[\delta](z_1-z_2)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)}\,, \tag{3.7}$$

Here $\delta \equiv (\alpha,\beta)$ encodes the spin structure of the various contributions and $\langle\delta|\gamma\rangle = e^{4\pi i(\alpha\gamma_2-\beta\gamma_1)}$ encodes the GSO phases [10].

Let us first, following [7], take $z_1+z_2 = 2\Delta_\gamma$, that is place z_1+z_2 at a divisor corresponding to the canonical class, without setting $z_1=z_2$. The contribution (3.7) then simplifies to

$$\sum_\delta \langle\delta|\gamma\rangle \theta[\delta]^3(0)\theta[\delta](z_1-z_2) = 4\theta[\gamma]^4(z_1-\Delta_\gamma)\,, \tag{3.8}$$

where in the last step we have used a Riemann identity. The Riemann Vanishing Theorem then implies that this vanishes identically as a function of z_1. Thus in this case whatever poles arise as $z_1\to z_2$, the identical zero from the spin structure sum cancels it.

Now turning to the ghost piece of the correlation function of picture-changing operators, one obtains contributions isomorphic to (3.8) as well as

$$\omega_i(z_1)\frac{\theta[\delta]^5(0)\partial_i\theta[\delta](2z_2-2\Delta_\gamma)}{\theta^2[\delta](z_1+z_2-2\Delta_\gamma)}\,, \tag{3.9}$$

Here ω_i are the canonical basis of holomorphic one-forms on the Riemann surface, satisfying $\int_{a_i}\omega_j = \delta_{ij}$ and $\int_{b_i}\omega_j = \tau_{ij}$ where τ is the period matrix for the surface. Again simplifying this by first taking $z_1+z_2 = 2\Delta_\gamma$ we obtain

$$\sum_\delta \langle\delta|\gamma\rangle \partial_{z_1}\left(\theta[\delta]^3(0)\theta[\delta](z_1-z_2)\right) = 4\partial_{z_1}\left(\theta[\gamma]^4(z_1-\Delta_\gamma)\right), \tag{3.10}$$

Because the right-hand side of this expression is a derivative of 0 (by the Riemann vanishing theorem), it vanishes identically. Again any poles from the picture changing OPEs are irrelevant [7].

4. 2-loop cancellation in F, G model

Let us consider the left-moving spin-structure-dependent pieces of the 2-loop partition function in the model of section 2. As explained in [1], all supersymmetry-breaking twist structures map by modular transformations to a canonical one in which there are twists $(1,1,F,G)$ in going around cycles (a_1,a_2,b_1,b_2). The F twist, which just involves an action of $(-1)^{F_R}$ and a symmetric shift, does not affect the θ-function characteristics and phases. The effect of the G twist is via its $(-1)^{F_L}$ action. This changes the G eigenvalues by a factor of (-1) in the left-moving Ramond sector relative to a model without the $(-1)^{F_L}$. In our twist structure there is a G operator inserted on the second handle. So the effect of the G twist is to change the phase of terms in the spin structure sum with $\alpha_2 = 1/2$, that is terms with Ramond sector states propagating around handle 2. This means that instead of phases $\langle\delta|\gamma\rangle$ we should have phases $\langle\delta|\gamma + (0,0,0,1/2)\rangle$. We will set $\kappa \equiv \gamma + (0,0,0,1/2)$ to simplify the formulas.

Then after imposing $z_1 + z_2 = 2\Delta_\gamma$, we get for the matter correlator

$$\sum_\delta \langle\delta|\kappa\rangle \theta[\delta]^3(0)\theta[\delta](z_1 - z_2)\theta[\gamma]\left(\sum_{i=1}^{3} w_i - 3\Delta_\gamma\right)\frac{\prod_{i<j} E(w_i,w_j)}{E(z_1,z_2)^2} \times$$
$$\times \left(2\pi p_i^\mu p_j^\mu \omega_i(z_1)\omega_j(z_2) + 10\,\partial_{z_1}\partial_{z_2}\log E(z_1,z_2)\,,\right) \tag{4.1}$$

where $E(z_1,z_2)$ is the prime form, which has the property that it goes like $z_1 - z_2$ as $z_1 \to z_2$. Here the $w_i, i = 1,\ldots,3h-3$ are the insertion points for the b-ghosts. In the full amplitude [11], they are contracted with Beltrami differentials encoding the gauge choice for the worldsheet metric. The spin-structure-dependent pieces here (i.e. those that depend on the characteristic δ) sum up to

$$4\theta[\kappa](z_1 - \Delta_\gamma)^4\,. \tag{4.2}$$

Because of the shift in the theta characteristic after the spin structure sum, the right hand side no longer vanishes identically. However, if we choose the reference spin structure γ such that both γ and $\kappa \equiv \gamma + (0,0,0,1/2)$ are odd, then we can impose $z_1, z_2 \to \Delta_\gamma$ and the right hand side of (4.2) vanishes like $(z_1 - z_2)^4$ as $z_1 \to z_2$. As $z_1 \to z_2$ the determinant factor (3.4) produces another zero: plugging in the delta function η_a we obtain

$$[sdet(\eta,\Phi)]^{-1} \propto \det\left(\eta_a, \Phi_b^{3/2}\right) = \det\left(\Phi_b^{3/2}(z_a)\right). \tag{4.3}$$

Here $\Phi_b^{3/2}, b = 1,2$ form a basis of holomorphic 3/2-differentials. As the z_a approach each other, the determinant (4.3) goes to zero.

Combining the spin structure sum with the determinant factor we have a fifth order zero. As we take the limit $z_1 \to z_2$ we must take into account the poles arising from the prime form $E(z_1,z_2)$ (which goes like $z_1 - z_2$ as $z_1 \to z_2$). The $2\pi p_i^\mu p_j^\mu \omega_i(z_1)\omega_j(z_2)$ term in (4.1) has a second order pole, which is beaten by our fifth order zero. The other contributions yield

$$\det\left(\eta_a, \Phi_b^{3/2}\right) 4\theta[\kappa]\left(\frac{1}{2}(z_1 - z_2)\right)^4 \theta[\gamma]\left(\sum w - 3\Delta_\gamma\right)\prod_{i<j} E(w_i,w_j)\frac{10}{(z_1 - z_2)^4}\,. \tag{4.4}$$

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The poles arise from the correlator of picture changing operators. The leading singularity in this correlator is proportional to $\frac{c}{(z_1-z_2)^4}$ where c is the total central charge (matter+ghost). Since this vanishes, there should be no such contribution after including the ghost terms in the picture changing operators. We will see this cancellation after computing the ghost correlators. In any case our fifth order zero suffices to cancel this contribution even before including the ghost piece, but after doing the spin structure sum.

The ghost correlator

$$\left\langle -\frac{1}{4}c\partial\xi(z_1)\left(2\partial\eta e^{2\phi}b+\eta\partial e^{2\phi}b+\eta e^{2\phi}\partial b\right)(z_2)\right\rangle + \\ + \left\langle -\frac{1}{4}\left(2\partial\eta e^{2\phi}b+\eta\partial e^{2\phi}b+\eta e^{2\phi}\partial b\right)(z_1)c\partial\xi(z_2)\right\rangle . \tag{4.5}$$

gives rise to three types of contributions (see [11] eqs. (38-39) for the complete expression). One is

$$\sum_\delta \langle\kappa|\delta\rangle \frac{\theta[\delta](0)^5\theta[\delta](2z_2-2\Delta_\gamma)\omega^i(z_1)\partial^i\theta[\gamma]\left(z_1-z_2+\sum w-3\Delta_\gamma\right)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2}\frac{\prod_{i<j}E(w_i,w_j)}{E(z_1,z_2)^3}\,. \tag{4.6}$$

Setting $z_1+z_2=2\Delta_\gamma$ and summing over δ we get for the spin-structure-dependent piece:

$$4\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)^4 \sim (z_1-z_2)^4\,. \tag{4.7}$$

In this contribution, there is a third-order pole from the prime forms, but a fifth-order zero (4.7) from the spin structure sum times the determinant factor (4.3), so the contribution cancels as $z_1\to z_2$.

The second ghost contribution takes the form

$$\sum_\delta \langle\kappa|\delta\rangle \frac{\theta[\delta](0)^5\theta[\delta]\left(2z_2-2\Delta_\gamma\right)\theta[\gamma]\left(z_1-z_2+\sum w-3\Delta_\gamma\right)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2}\times \\ \times\frac{\prod_{i<j}E(w_i,w_j)}{E(z_1,z_2)^3}\partial_{z_1}\log\left(\frac{\prod_j E(z_1,w_i)}{E(z_1,z_2)^5\sigma(z_1)}\right)+(z_1\leftrightarrow z_2)\,, \tag{4.8}$$

where $\sigma(z_1)$ is defined in [11] and has no zeroes or poles. Taking $z_1+z_2=2\Delta_\gamma$ and summing over δ we get

$$4\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)^4\frac{\prod_{i<j}E(w_i,w_j)}{E(z_1,z_2)^3}\partial_{z_1}\log\left(\frac{\prod_j E(z_1,w_i)}{E(z_1,z_2)^5\sigma(z_1)}\right)+(z_1\leftrightarrow z_2)\,. \tag{4.9}$$

This contribution has again a fifth order zero from the spin-structure sum times the determinant (as in (4.7)). There is also a fourth-order pole from the prime forms. The contribution is

$$\det\left(\Phi_b^{3/2}(z_a)\right)4\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)^4\prod_{i<j}E(w_i,w_j)\frac{-5}{(z_1-z_2)^4}\theta[\gamma]\left(\sum w-3\Delta_\gamma\right)+(z_1\leftrightarrow z_2). \tag{4.10}$$

This piece cancels against the matter contribution (4.4). As mentioned above this en-

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codes the fact that the total central charge vanishes. Again taking into account (4.3) we in fact obtain cancellation directly.

Now let us consider the remaining ghost contributions. These arise from the correlator

$$\left\langle c\partial\xi(z_1)\left(\partial e^{2\phi}\right)\eta b(z_2)\right\rangle + \left\langle\left(\partial e^{2\phi}\right)\eta b(z_1)\; c\partial\xi(z_2)\right\rangle . \tag{4.11}$$

They take the form

$$\det\left(\Phi_b^{3/2}(z_a)\right)\sum_\delta \langle\kappa|\delta\rangle\;\frac{\theta[\delta](0)^5\omega_i(z_1)\partial^i\theta[\delta](2z_2-2\Delta_\gamma)\theta[\gamma](z_1-z_2+\sum w-3\Delta_\gamma)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2}\times$$
$$\times\frac{\prod_{i<j}E(w_i,w_j)}{E(z_1,z_2)^3}+(z_1\leftrightarrow z_2)\,. \tag{4.12}$$

In the first term of this contribution (4.11), setting $z_1+z_2=2\Delta_\gamma$ and summing on δ gives

$$\omega_i(\Delta_\gamma)\partial^i\theta[\kappa](z_1-z_2)^4\frac{1}{(z_1-z_2)^3}\,, \tag{4.13}$$

plus other terms which have lower-order poles. The spin structure sum only gives a third order zero here due to the derivative acting on the theta functions. The $\det\left(\Phi_b^{3/2}(z_a)\right)$ factor gives an additional zero which kills this contribution as $z_1\to z_2$.

Thus we see that in the (F,G) model we can pick a gauge for the worldsheet metric as well as gravitino, for which all the contributions cancel, after summing over all spin structures and using appropriate Riemann identities. Note that the cancellations explained above occur via the spin structure sum of each term in the correlator of picture-changing operators separately, so that the shifts twisting some of the bosons do not affect the argument. The above cancellation alone is not enough to guarantee that the two-loop amplitude cancels; one needs the cancellation of the boundary contributions that potentially arise due to the ambiguity in the choice of gauge slice for the gravitini [18]. In the (F,G) model these appear to cancel similarly to the analyis in the original models [1]. Note that although the bulk cancellations we exhibit here would hold also for example for the type of model studied in [19] (generated by a translation combined with $(-1)^{F_L+F_R}$), those models have a nonzero one-loop vacuum energy and non-zero one-loop tadpoles (as well as tachyons, for some range of the moduli VEVs). Therefore, the boundary contributions in the latter case would destroy the cancellation. Nonetheless it is somewhat surprising that the bulk contribution can be made to cancel by choosing an appropriate gauge in that model.

5. Taylor expansion in original models

In this section we will present the analysis of the coincident gauge for the original models of [1] (see that paper for the definition of the model). As in [1], we only need to analyze the twist structure $(1,1,f,g)$. The f twist affects the characteristics of

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some of the θ functions (arising from twisted fields) by shifting them by $(0,0,1/2,0)$ – we shall denote this as a shift by $\frac{1}{2}L$. κ is, as in §4, $\gamma+(0,0,0,1/2)$, and we choose γ such that both γ and κ are odd.

The correlation function of the matter part of the picture changing operators breaks into two contributions. The terms involving $\langle\psi^i\partial X^i(z_1)\psi^i\partial X^i(z_2)\rangle$ with $i=5,\ldots 10$ give

$$\sum_\delta\langle\kappa|\delta\rangle\frac{\theta[\delta](0)^2\theta[\delta+\frac{1}{2}L](0)^2\theta[\delta](z_1-z_2)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)}\times \\ \times\left(p_i^\mu\omega^i(z_1)p_j^\mu\omega^j(z_2)\frac{1}{E(z_1,z_2)^2}+\frac{6}{E(z_1,z_2)^2}\partial_{z_1}\partial_{z_2}\log E(z_1,z_2)\right)\det\left(\Phi_a^{3/2}(z_b)\right). \tag{5.1}$$

Upon setting $z_1+z_2=2\Delta_\gamma$ we can cancel the denominator against one factor in the numerator to get

$$\sum_\delta\langle\kappa|\delta\rangle\theta[\delta](0)\ \theta\ [\delta](z_1-z_2)\theta\left[\delta+\frac{1}{2}L\right](0)^2= \\ =4\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)\theta\left[\kappa+\frac{1}{2}L\right]\left(\frac{1}{2}(z_1-z_2)\right), \tag{5.2}$$

for the spin-dependent piece of this correlator. Because κ is an odd spin structure, this vanishes like $(z_1-z_2)^2$. The factor of $\det\left(\Phi_a^{3/2}(z_b)\right)$ in (5.2) also vanishes like (z_1-z_2) as $z_1\to z_2$, so all in all (5.2) has a $(z_1-z_2)^3$ multiplying the prime forms. However, since the prime-forms are yielding poles as $z_1\to z_2$, it remains to check that there are no finite pieces in (5.2).

The terms proportional to $\frac{1}{E(z_1,z_2)^2}$ times the loop momenta clearly vanish in the limit, since there is only a second order pole from the prime forms which cannot cancel the third order zero we found from the spin structure sum and the superdeterminant. This leaves the term which goes like $\frac{1}{E(z_1,z_2)^2}\partial_{z_1}\partial_{z_2}\log E(z_1,z_2)$. Using the fact that $E(z_1,z_2)$ has a Taylor expansion of the form

$$E(z_1,z_2)\sim\sum_{n=0}^{\infty}c_n(z_1-z_2)^{2n+1}, \tag{5.3}$$

as $z_1\to z_2$, one sees that this combination of prime forms has an expansion

$$\frac{1}{E(z_1,z_2)^2}\partial_{z_1}\partial_{z_2}\log E(z_1,z_2)\sim\sum_{n=-2}^{\infty}d_n(z_1-z_2)^{2n}. \tag{5.4}$$

On the other hand, the determinant factor is an *odd* function of z_1-z_2 with an expansion of the form

$$\det\left(\Phi_a^{3/2}(z_b)\right)\sim\sum_{m=0}^{\infty}e_m(z_1-z_2)^{2m+1}, \tag{5.5}$$

while the sum over spin structures (5.2) is an even function with a second order zero at $z_1 = z_2$. From these facts, it is easy to see that the full expression (5.2) has an expansion of the form

$$\sum_{j=0}^{\infty} f_j (z_1 - z_2)^{2j-1}\,, \tag{5.6}$$

as $z_1 \to z_2$.

Examining (5.6), we see that

1. There are no finite contributions as $z_1 \to z_2$.

2. There is a (gauge artifact) pole as $z_1 \to z_2$; in fact this pole receives contributions from the various matter and ghost correlators proportional to the matter/ghost central charges, and hence cancels once all of the terms are taken into account (since $c_{tot} = c_{matter} + c_{ghost} = 0$). We will see this explicitly once we compute the remaining matter and ghost contributions.

The second type of matter correlator arises from contracting $\psi^i \partial X^i(z_1)\psi^i\partial X^i(z_2)$ with $i = 1, \ldots 4$. This leads to a contribution

$$\begin{aligned}&\sum_\delta \langle\kappa|\delta\rangle \frac{\theta[\delta](0)^3\theta[\delta+\frac{1}{2}L](0)\theta[\delta+\frac{1}{2}L](z_1-z_2)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)} \times \\ &\times \left(p_i^\mu\omega^i(z_1)p_j^\mu\omega^j(z_2)\frac{1}{E(z_1,z_2)^2} + \frac{4}{E(z_1,z_2)^2}\partial_{z_1}\partial_{z_2}\log E(z_1,z_2)\right)\det\left(\Phi_a^{3/2}(z_b)\right).\end{aligned} \tag{5.7}$$

Choosing $z_1 + z_2 = 2\Delta_\gamma$, the spin sum in (5.8) simplifies to

$$\sum_\delta \langle\kappa|\delta\rangle\theta[\delta](0)^2\theta\left[\delta+\frac{1}{2}L\right](0)\theta\left[\delta+\frac{1}{2}L\right](z_1-z_2)\,, \tag{5.8}$$

which, after applying a Riemann identity, becomes

$$4\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)^2\theta\left[\kappa+\frac{1}{2}L\right]\left(\frac{1}{2}(z_1-z_2)\right)^2. \tag{5.9}$$

So in fact after summing over spin structures this looks the same as the spin sum of the first type of matter contribution (5.2). Again, it vanishes like $(z_1 - z_2)^2$ as $z_1 \to z_2$.

Now, the argument for the cancellation proceeds as it did for the first type of matter contribution. The terms involving only the $\frac{1}{E(z_1,z_2)^2}$ multiplying loop momenta only have a second order pole, which cannot cancel the third order zero coming from the determinant times the spin structure sum (5.9). The terms involving higher inverse powers of the prime forms lead to a simple pole (which cancels after summing over matter and ghosts, as it is proportional to the total central charge) and no finite contributions.

Next, let us consider the terms in the correlator of picture changing operators coming from the ghost part of the worldsheet supercurrent (4.5). There are three types of terms that arise [11]. We are in the twist structure $(1,1,f,g)$. As in the matter sector, the f twist affects the characteristics of the θ-functions arising in the worldsheet correlation functions and determinants. We will denote the shift in the characteristic, which is $(0,0,1/2,0)$, as $\frac{1}{2}L$. The first type of contribution is

$$\sum_\delta \langle\kappa|\delta\rangle \frac{\theta[\delta](0)^3\theta[\delta+\frac{1}{2}L](0)^2\theta[\delta](2z_2-2\Delta_\gamma)\theta(z_1-z_2+\sum w-3\Delta)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2E(z_1,z_2)^3}\times$$
$$\times \det\left(\Phi_a(z_b)\right)\frac{\prod E(z_1,w)}{\prod E(z_2,w)}\partial_{z_1}\log\left(\frac{\prod E(z_1,w)}{E(z_1,z_2)^5\sigma(z_1)}\right)+(z_1\leftrightarrow z_2)\,. \tag{5.10}$$

The second is

$$\sum_\delta \langle\kappa|\delta\rangle \frac{\theta[\delta](0)^3\theta\left[\delta+\frac{1}{2}L\right](0)^2\omega_i(z_1)\partial^i\theta[\delta](2z_2-2\Delta_\gamma)\theta(z_1-z_2+\sum w-3\Delta)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2E(z_1,z_2)^3}\times$$
$$\times \det\left(\Phi_a(z_b)\right)\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+(z_1\leftrightarrow z_2)\,. \tag{5.11}$$

The third is

$$\sum_\delta \langle\kappa|\delta\rangle \frac{\theta[\delta](0)^3\theta[\delta+\frac{1}{2}L](0)^2\theta[\delta](2z_2-2\Delta_\gamma)\omega_i(z_1)\partial^i\theta(z_1-z_2+\sum w-3\Delta)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2E(z_1,z_2)^3}\times$$
$$\times \det\left(\Phi_a(z_b)\right)\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+(z_1\leftrightarrow z_2)\,. \tag{5.12}$$

Setting $z_1+z_2=2\Delta_\gamma$ and doing the spin structure sum we find for the spin-structure-dependent pieces of contributions (5.10) and (5.12):

$$\sum_\delta\langle\kappa|\delta\rangle\theta[\delta](0)\theta\left[\delta+\frac{1}{2}L\right](0)^2\theta[\delta](2z_2-2\Delta_\gamma)=\theta[\kappa](z_2-\Delta_\gamma)^2\theta\left[\kappa+\frac{1}{2}L\right](z_2-\Delta_\gamma)^2$$
$$\sim(z_1-z_2)^2+c_4(z_1-z_2)^4+\cdots \tag{5.13}$$

for some constant c_4 where in the last line we expanded the result in a Taylor expansion around $z_1=z_2$. For contribution (5.11) we get

$$\sum_\delta\langle\kappa|\delta\rangle\theta[\delta](0)\ \ \theta\ \left[\delta+\frac{1}{2}L\right](0)^2\omega_i(z_1)\partial^i\theta[\delta](2z_2-2\Delta_\gamma)=$$
$$=\partial_{z_1}\left(\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)^2\theta\left[\kappa+\frac{1}{2}L\right]\left(\frac{1}{2}(z_1-z_2)\right)^2\right)\sim$$
$$\sim(z_1-z_2)+b_3(z_1-z_2)^3+\cdots \tag{5.14}$$

As for the matter contributions, although the spin structure sums give vanishing contributions, they multiply singularities arising from the prime forms $E(z_1,z_2)$ and we must analyze the potential finite terms in the Taylor expansion. Let us consider first (5.10). There are two types of contributions here. After doing the spin structure sum as above the first takes the form

$$-5\frac{\partial_1 E(z_1,z_2)}{E(z_1,z_2)^4}\left[z_{12}^2+c_4 z_{12}^4+\cdots\right]\left[z_{12}+e_3 z_{12}^3+\cdots\right]\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+(z_1\leftrightarrow z_2)\,, \quad (5.15)$$

where we denote z_1-z_2 by z_{12}. Here the second factor comes from the spin structure sum, the third from the Taylor expansion of the determinant about $z_1=z_2$ (where e_3 is some constant). We should emphasize what is meant here by $(z_1\leftrightarrow z_2)$. We are computing a correlation function of picture changing operators. The ghost piece of this correlator has the form (4.5). So for example the second term in (4.5) corresponds to the term denoted $z_1\leftrightarrow z_2$ in (5.15). So in particular the second term involves interchanging the operators in the ghost correlator, without changing z_1 to z_2 in the determinant factor. The first and fourth factors involving the prime forms encode the physical poles and zeroes of the correlator. The leading singularity from the prime forms here comes from the $1/z_{12}^4$ term in the expansion of the prime form factors. Therefore only the leading term in the Taylor expansion of the spin structure sum and determinant factors potentially survive (so we can ignore the terms proportional to c_4 or e_3, which give fifth-order zeroes). Similarly expanding the prime forms $E(z_1,z_2)$ gives a subleading term with only a $1/z_{12}^2$ pole, which is cancelled by the third order zero coming from the leading piece of the spin structure sum times determinant.

Putting the factors together, we see that the leading piece is a simple pole in z_{12}. The first three factors in (5.15) are the same in the term with $z_1\leftrightarrow z_2$. When we include the term with $z_1\leftrightarrow z_2$, they multiply the prime form factor $\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+\frac{\prod E(z_2,w)}{\prod E(z_1,w)}$. This is even under $z_1\leftrightarrow z_2$. In our Taylor expansion it therefore becomes of the form $\mathcal{O}(1)+f_2 z_{12}^2+\cdots$, and only the first term contributes. Therefore in Taylor expanding the contribution (5.15), we get a pole piece plus higher order terms which vanish in the limit $z_1\to z_2$. In particular, no finite pieces survive. What is the interpretation of the pole piece? It is proportional to the ghost central charge, and precisely cancels the pole piece coming from the matter contribution.

The second type of contribution in (5.10) takes the form

$$\frac{1}{E(z_1,z_2)^3}[z_{12}^2+\cdots][z_{12}+\cdots]\frac{\prod E(z_1,w)}{\prod E(z_2,w)}\partial_1\log\left(\frac{\prod E(z_1,w)}{\sigma(z_1)}\right), \quad (5.16)$$

where the $\cdots$ denote terms which vanish automatically as $z_1\to z_2$. The leading pole from the prime forms here is cubic. Before including the $z_1\leftrightarrow z_2$ term there is a finite piece obtained by multiplying this times the third order zero obtained from the spin structure sum and determinant factors. The spin structure sum is even under the interchange of z_1 and z_2 in this case, and as discussed above the determinant factor is the same in both terms. The factor $\frac{1}{E(z_1,z_2)^3}$ does change sign between the two terms, however. So when we add the $(z_1\leftrightarrow z_2)$ term the contribution cancels.

Let us now consider the contribution (5.11). This gives a contribution of the form

$$\frac{1}{E(z_1,z_2)^3}[z_{12}+\cdots][z_{12}+\cdots]\left(\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+\frac{\prod E(z_2,w)}{\prod E(z_1,w)}\right). \tag{5.17}$$

Here similarly to the above analysis we took into account the relative sign of the two contributions in (4.5) and included the $z_1 \leftrightarrow z_2$ contribution. The last factor here is even under interchange of z_1 and z_2, so its Taylor expansion is of the form $1 + h_2 z_{12}^2 + \cdots$ for some constant h_2. The leading contribution here is a simple pole, and there is no finite contribution.

Unlike the previous simple poles we have encountered, the pole encountered here does not cancel with the other matter and ghost contributions (it is *not* one of the pieces which would have contributed to the $\frac{c}{z^4}$ pole in the OPE of picture changing operators before accounting for spin structure sums and determinant factors). However, on general grounds we expect such gauge artifact poles to constitute total derivatives on moduli space. Otherwise the invariance of the path integral on gauge slice would be lost. In this case, we can argue for that conclusion as follows. The pole we are discussing receives a $\frac{1}{(z_1-z_2)^3}$ contribution from the prime forms which is softened to $\frac{1}{(z_1-z_2)^2}$ by the theta function zero (and then to a simple pole by the determinant factor). In the OPE of picture changing operators, the $\frac{1}{(z_1-z_2)^2}$ divergence is multiplied by the stress-energy tensor, which gives a derivative with respect to the metric and therefore the moduli. The term we are finding is part of this total derivative. In the gauge we have chosen, it is the only non-vanishing piece (the other pieces vanish even before integration over the moduli space). However, since there cannot be gauge artifact poles, we expect it to integrate to zero (which one can argue for using the boundary analysis in [1]).

Finally let us consider the last ghost contribution (5.12). This contribution takes the form

$$\frac{1}{E(z_1,z_2)^3}[z_{12}^2+\cdots][z_{12}+\cdots]\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+(z_1\leftrightarrow z_2). \tag{5.18}$$

In this contribution before including the $z_1 \leftrightarrow z_2$ contribution there is a potential finite term from the third order pole multiplying a third order zero in z_{12}. Here again, in the limit $z_1 \to z_2$ every factor except the first is the same in the two terms. The first factor $\frac{1}{E(z_1,z_2)^3}$ has the opposite sign in the two terms. Thus again after including the $z_1 \leftrightarrow z_2$ term the contribution cancels.

6. Discussion

In the original models of the type described in [1], as well as in the new model described in [2], we see here that the two loop contribution cancels in the gauge with coincident picture changing operators after properly taking into account the singularities.

This analysis, combined with the analysis of boundary contributions presented in [1], yields a vanishing two loop cosmological constant in these models. Note that we computed this contribution in a particular twist structure, in the gauge where z_1 and z_2 approach a particular branch point Δ_γ. Performing a modular transformation moves us to a different twist structure–as explained in [1] one can recover all twist structures in this way–but also permutes the branch points and in general will change the gauge choice.[3] However at a given twist structure, one choice of insertion points for picture changing operators is related to another by total derivative terms which cancel in our model [1]. Therefore in a fixed gauge we are in fact not finding cancellation pointwise on the moduli space, but instead only after integration (using the formal argument of [11] to reduce the integrals to boundary contributions). A modular invariant integrand is obtained on the other hand if we let the gauge choice get modular tranformed along with the twist structure. Needless to say it would be interesting to compute the integral directly in some fixed gauge to check the cancellation independently.

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